HSRGC Colloquium

Testing GR with Shadows and Quasinormal Modes

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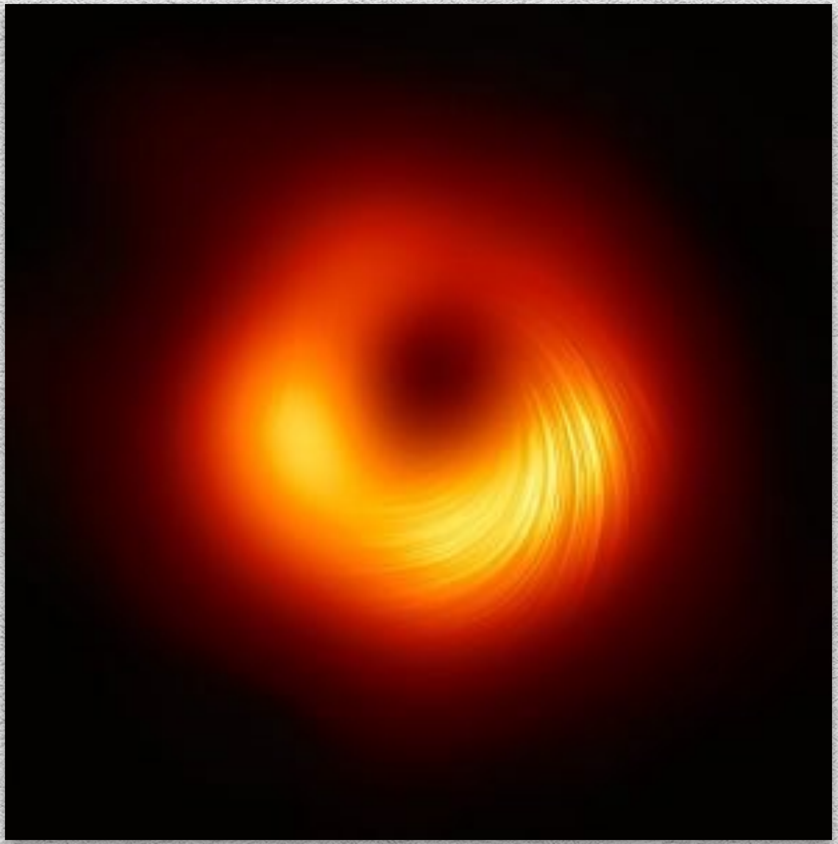
April 26, 2021

Abstract

In the first part of this talk we summarise some recent progress in modelling quasinormal modes of non-GR black holes. In the second part we address the issue of testing GR with supermassive black hole shadows.

Testing GR with black hole shadows & quasinormal modes



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Part I

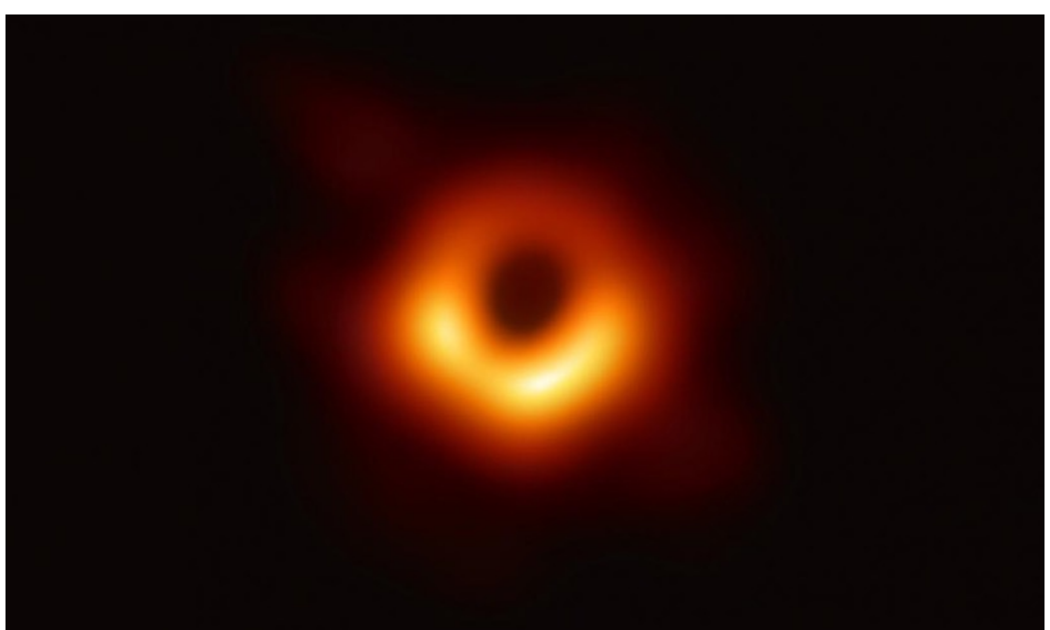
Can we test gravity with supermassive black hole shadows?

Based on KG & Pappas arXiv:2102.13573

A century apart ...

1919

The Eddington-Dyson solar eclipse expeditions measure gravitational deflection of light, thus bolstering confidence to Einstein's recently formulated GR theory.



2019

The EHT collaboration releases the first direct image of a BH, the supermassive BH at the centre of the M87 galaxy. The shadow (associated with the BH's photon ring) also represents extreme light deflection.

BH shadow and its radius

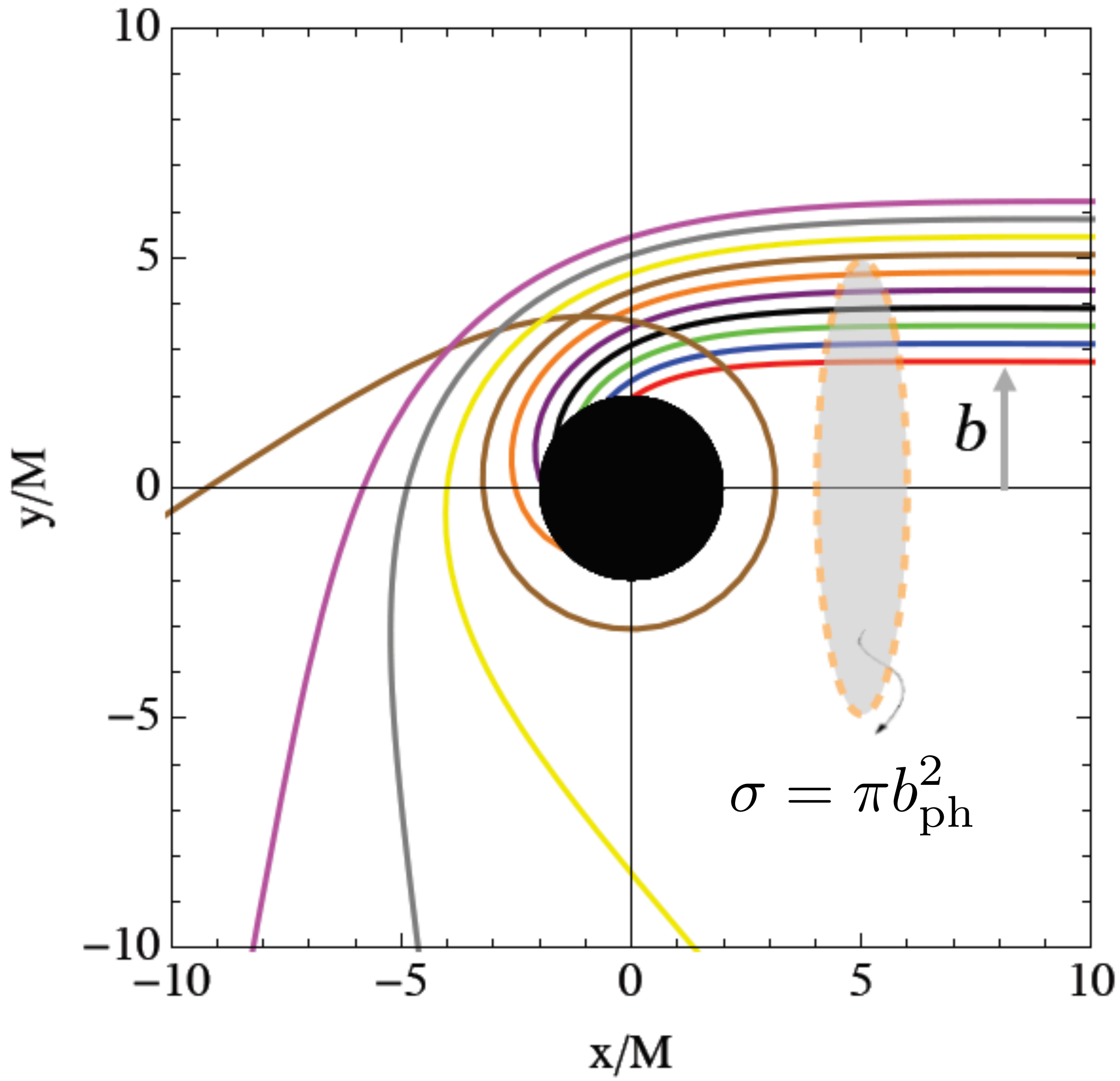
- The textbook definition of a BH shadow is summarised in the figure.

- The shadow is really a manifestation of (i) the presence of a photon ring and (ii) the absence of a light-emitting surface.

- The shadow radius is an intrinsic property of the spacetime, determined by the radius of the photon ring (unstable circular photon orbit).

$$b_{\rm ph} = 3\sqrt{3}M \approx 5.2M$$

Schwarzschild BHs



BH shadow and its radius

- The shadow of Kerr BHs is almost circular unless the spin is close to its maximum value a=M and the BH is viewed “edge on”.

One of the first published shadow figures, Bardeen (1972).

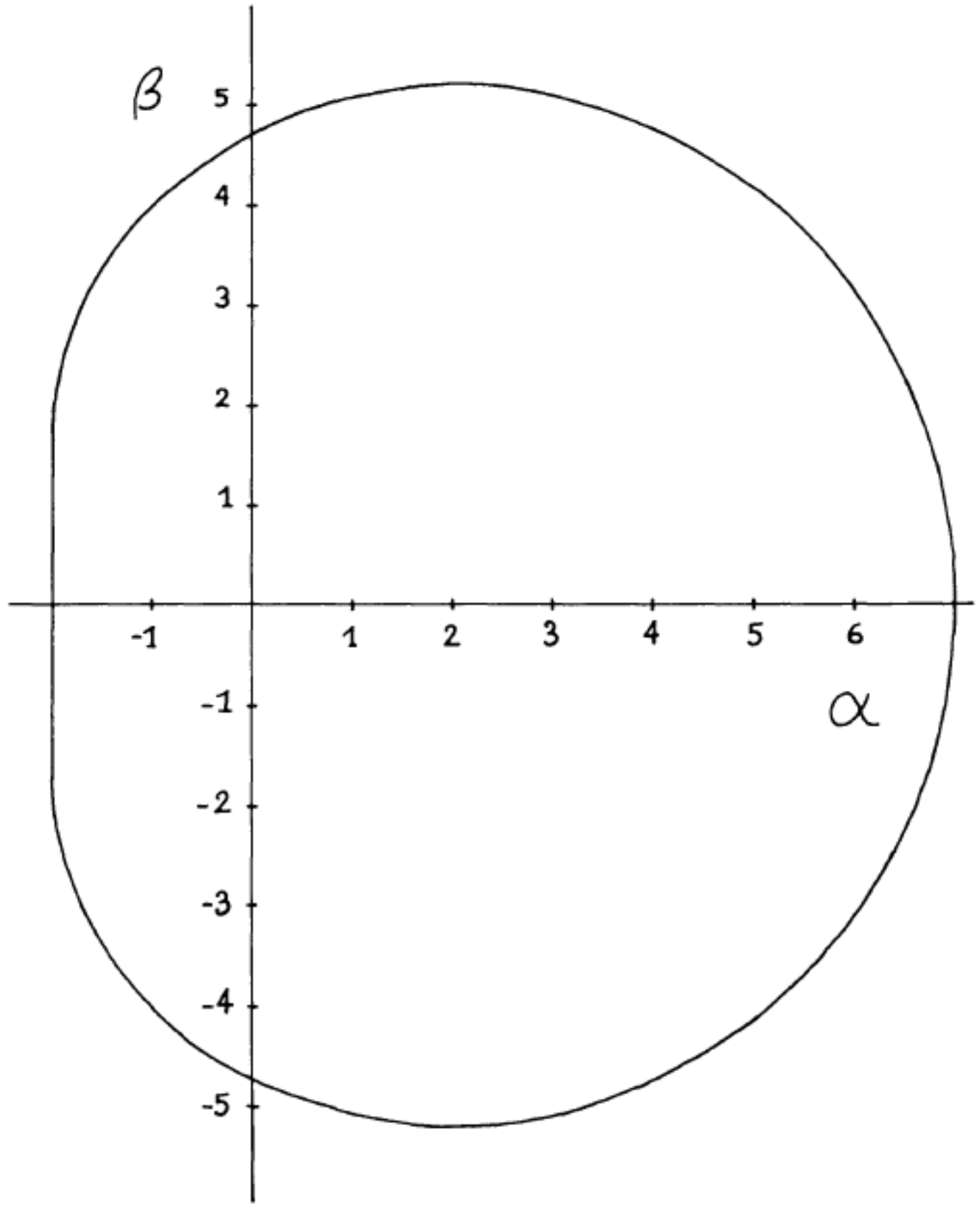


Figure 6. The apparent shape of an extreme ($a = m$) Kerr black hole as seen by a distant observer in the equatorial plane, if the black hole is in front of a source of illumination with an angular size larger than that of the black hole.

M87* black hole factsheet

$$M \approx 6.6 \times 10^9 \, M_\odot$$

Mass estimated from:

(i) The radius of the *quasi-circular* shadow, given the measured angular size/distance and assuming GR.

(ii) Stellar kinematics in the vicinity of the BH.

$$d \approx 17.9 \, \mathrm{Mpc}$$

$$\frac{J}{M^2} = \frac{a}{M} = \text{poorly constrained}$$

The accretion flow geometry (modelled with the help of numerous GR-MHD simulations) is uncertain, being something between quasi-spherical to a thick disk configuration.

The M87* shadow as a test of GR gravity (and of the "Kerr hypothesis")

- A recent EHT paper [Psaltis et al. PRL 125 (2020)] used the shadow radius to constrain deviations from GR.

- This was done with the help of the Johannsen (2013) metric, a cleverly designed parametrised deformed Kerr spacetime.

- For simplicity we ignore the BH spin (it only has a modest effect unless it is very high). The shadow radius can be identified with the impact parameter *b* of the photon ring. Only one metric component matters:

$$b = \frac{r_{\rm ph}}{\sqrt{-g_{tt}(r_{\rm ph})}}, \qquad r_{\rm ph}\frac{dg_{tt}}{dr}(r_{\rm ph}) = 2g_{tt}(r_{\rm ph})$$

$\varepsilon_3, \alpha_{13} =$ deformation parameters

$$g_{tt}(r) = -\left(1 - \frac{2M}{r}\right)\frac{(1+\varepsilon_3 M^3/r^3)}{(1+\alpha_{13}M^3/r^3)^2}$$

GR limit:

$\varepsilon_3 = \alpha_{13} = 0$

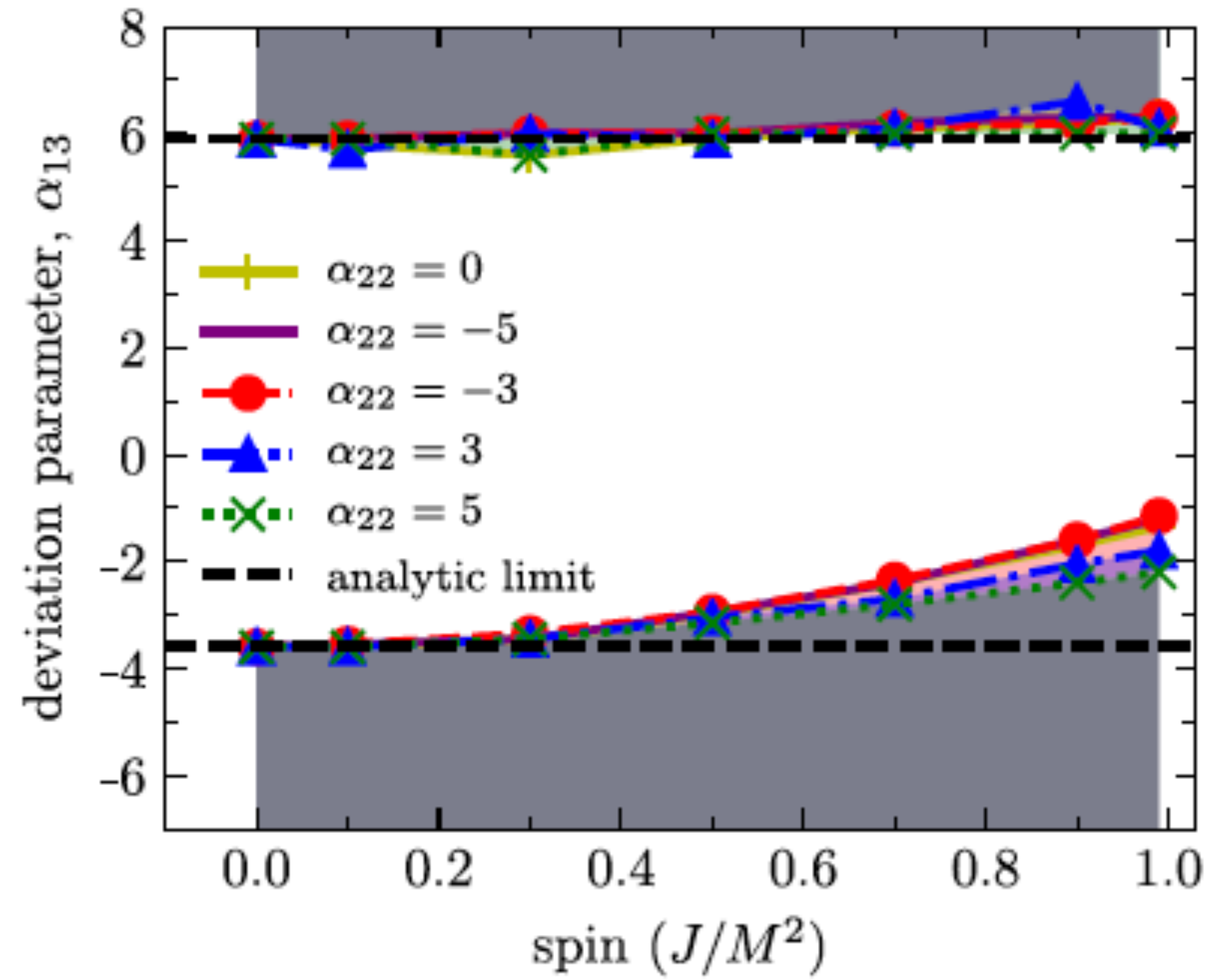
The M87* shadow as a test of GR

- The ≈ 17 % error in the shadow radius relative to the GR value

$$b_{\rm GR} = 3\sqrt{3}M \approx 5.2M$$

translates to an upper/lower bound for α_{13} (assuming $\varepsilon_3 = 0$).

- With more deformation parameters "switched on" these bounds (per parameter) become weaker.



Psaltis et al. (2020)

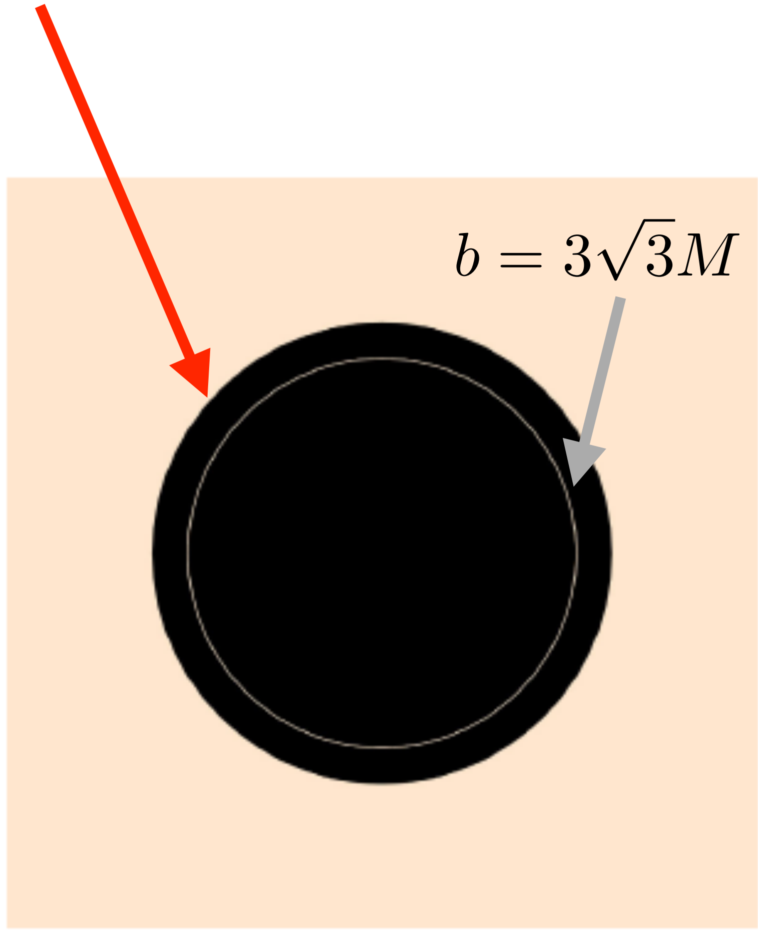
- However, such tests of GR come with some caveats:

Matter: the impact of the largely unknown accretion properties. [Gralla (2020)]

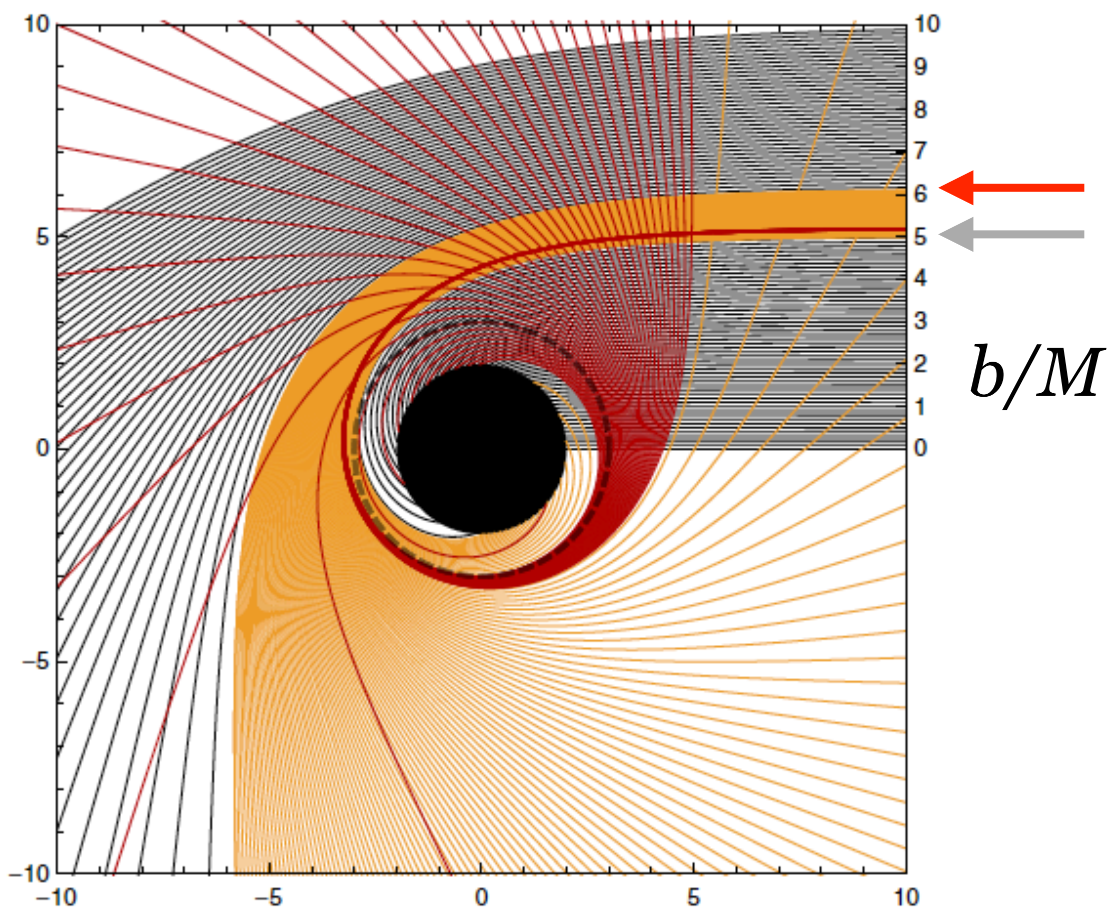
Gravity: *the impact of dimensional constants in non-GR theories.*

The impact of accretion physics

- The actual apparent shadow radius does depend on the geometry of the illuminating accretion flow. This can vary from a quasi-spherical flow to a thin disk flow.
- An (unrealistic) example: the shadow of a “backlit” BH is somewhat larger, $b \approx 6.2M$.



Gralla et al. (2019)



The impact of accretion physics

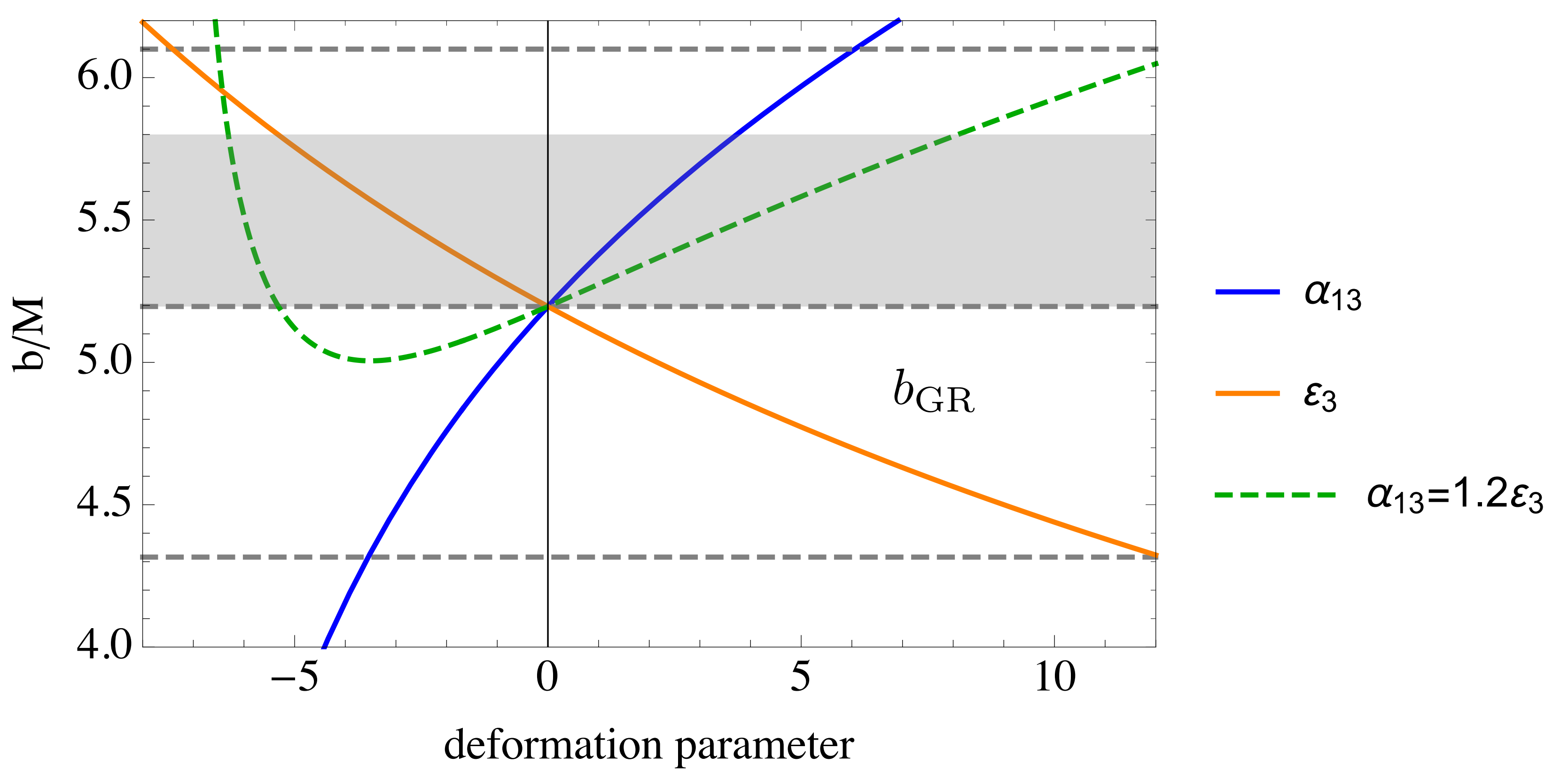
- Accretion in M87* is something between quasi-spherical to a thick disk configuration. The corresponding shadow radius is [Gralla et al. (2019)]:

$$5.2 \lesssim b/M \lesssim 5.8$$

- We can replot the previous shadow radius figure (assuming a non-rotating BH) as a function of the deformation away from GR, with the "matter uncertainty" added.

The uncertainty in the accretion physics mostly overlaps with the $b > b_{\rm GR}$ measurement error.

The $b < b_{\rm GR}$ range remains "clean".



Non-GR gravity: the key role of dimensional coupling constants

- Many of the widely studied alternative to GR theories of gravity are described by Lagrangians of the general form:

$$\mathcal{L} = \mathcal{L}_{\rm GR} + \mathcal{L}_{\rm scalar} + \alpha\{\text{scalar terms}\} \times \{\text{non-linear curvature terms}\} + \mathcal{L}_{\rm mat}$$

with a *coupling constant* of dimensionality: $\alpha = (\text{length})^n = (\text{mass})^n$ $n \geq 1$

- A typical example is Einstein-scalar-Gauss-Bonnet gravity (EsGB):

$$\mathcal{L} = \frac{1}{16\pi}\left[R - \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - \alpha f(\phi)\mathcal{R}^2_{\rm GB}\right] + \mathcal{L}_{\rm mat}$$

$f(\phi) =$ dimensionless, "user-specified"

with $\mathcal{R}^2_{\rm GB} = R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} - 4R^{\alpha\beta}R_{\alpha\beta} + R^2$

$\alpha = (\text{length})^2$

Bounds from GW signals of binary BHs

- Among other things, GWs probe the celestial mechanics of the binary.

- For the particular example of EdGB gravity, the metric of a non-rotating BH looks like this:

$$g_{tt} = -\left(1 - \frac{2M}{r}\right) + \frac{8}{3}\eta^2 \frac{M^3}{r^3}\left\{1 + \mathcal{O}\left(\frac{M}{r}\right)\right\} + \mathcal{O}(\eta^3) \qquad \eta = \frac{\alpha f'(\phi_\infty)}{4M^2}$$

[Julié & Berti (2019)]

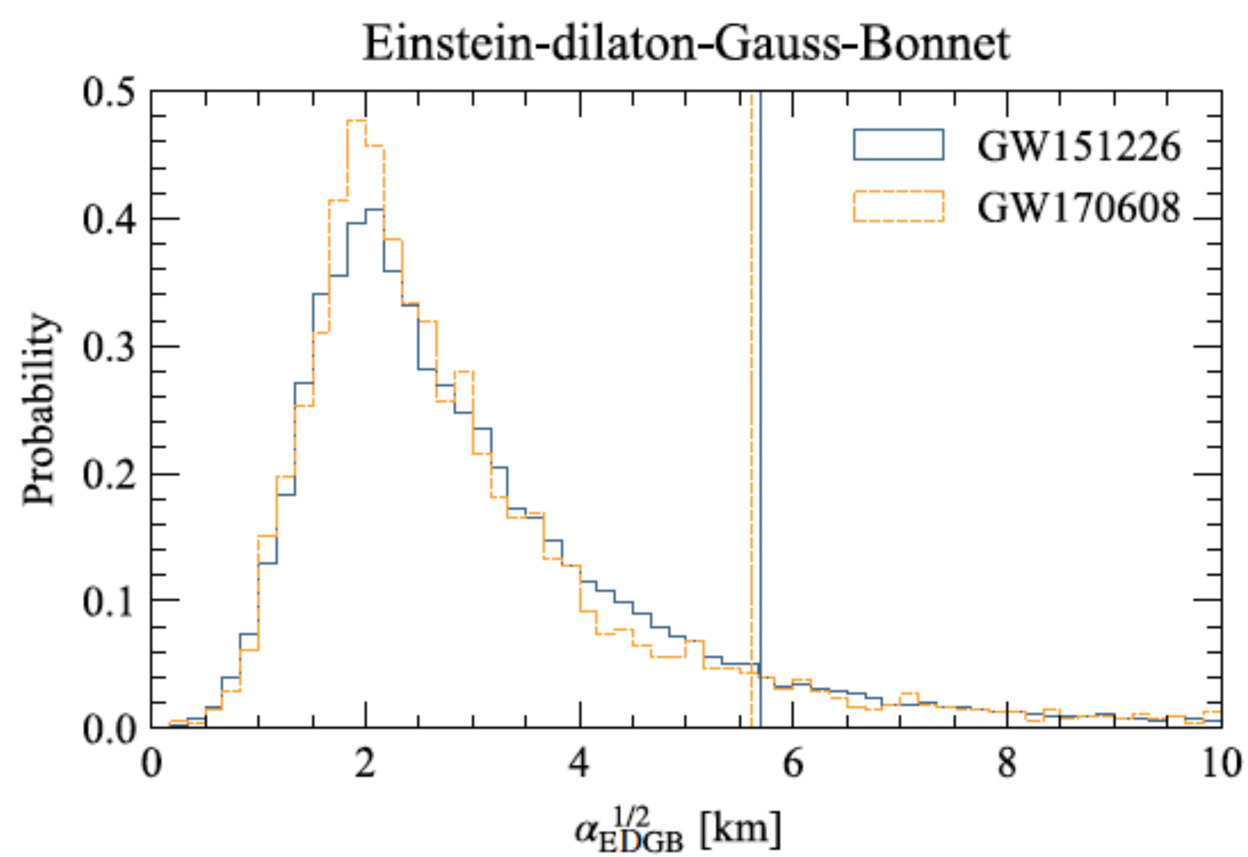
- $\eta > 1$ implies significant deviation from GR during the last few orbits of a binary BH.

- Bounds from LIGO/Virgo:

$$\eta_{\rm bin} \lesssim \mathcal{O}(1) \Rightarrow \alpha^{1/2} \lesssim \text{few km}$$

$$M_{\rm bin} \sim 100 M_\odot$$

Nair et al. (2019) + Clifton et al. (2020)



Mass-rescaled coupling constant

- Now assume we want to test a theory like EsGB using the M87* shadow.

- The shadow and the observed image is the result of photons moving along the BH's geodesics. We can write schematically:

EsGB geodesics- GR geodesics = O(η)

- The theory's extra "universal constant" is α, which is constrained by GW data. For a supermassive BH like M87* the dimensionless η is mass-rescaled:

$$\eta_{\rm M87} \sim \frac{\alpha}{M_{\rm M87}^2} \sim \eta_{\rm bin} \left(\frac{M_{\rm bin}}{M_{\rm M87}} \right)^2 \lesssim 10^{-14}$$

- Photons near M87* "see" a GR BH spacetime; *to a very high precision the shadow is indistinguishable from Kerr and the test fails*!

- The same argument applies to EMRI sources for LISA [Maselli et al. (2020)], but in that case the instrument also probes the non-geodesic orbital evolution.

Implications for shadow-based tests of GR

- The previous analysis, and given the current precision of GW observations, has serious repercussions for probing deviations from GR using the image/shadow of a supermassive BH.

- **Supermassive BH shadows *cannot* test non-GR theories when:**

★ The Kerr metric is still an admissible solution (trivial case).

★ The BH metric is non-Kerr but the theory is endowed with dimensional coupling constants. This class contains the majority of theories considered in the literature.

- **Supermassive BH shadows *can* test non-GR theories when:**

★ A theory has *dimensionless* coupling constants in its Lagrangian. A member of this minority class is Einstein-aether gravity.

★ A theory with dimensional coupling constants can *evade* the GW bounds in stellar mass BHs (see next slide).

Evading the GW bounds (I)

- There are two mechanisms that could enable the evasion of GW bounds from compact object binaries.

- *Screening*: the non-GR degrees of freedom are suppressed below some lengthscale (this is a “trick” commonly used in cosmology for evading bounds from solar system and compact binary tests). Typically, this is achieved by a scalar-coupling modification of the matter part of the Lagrangian.

$$\mathcal{L} = \mathcal{L}_{\rm GR} + \mathcal{L}_{\rm non-GR} + \mathcal{L}_{\rm mat}$$

This is irrelevant for BHs, given their vacuum solution nature.

- BH screening could be achieved by the addition of higher-order derivatives in the “non-GR” part of the Lagrangian (e.g. the Vainshtein mechanism). However, screening solar-mass BHs but not supermassive ones may require some fine-tuning in the screening physics.

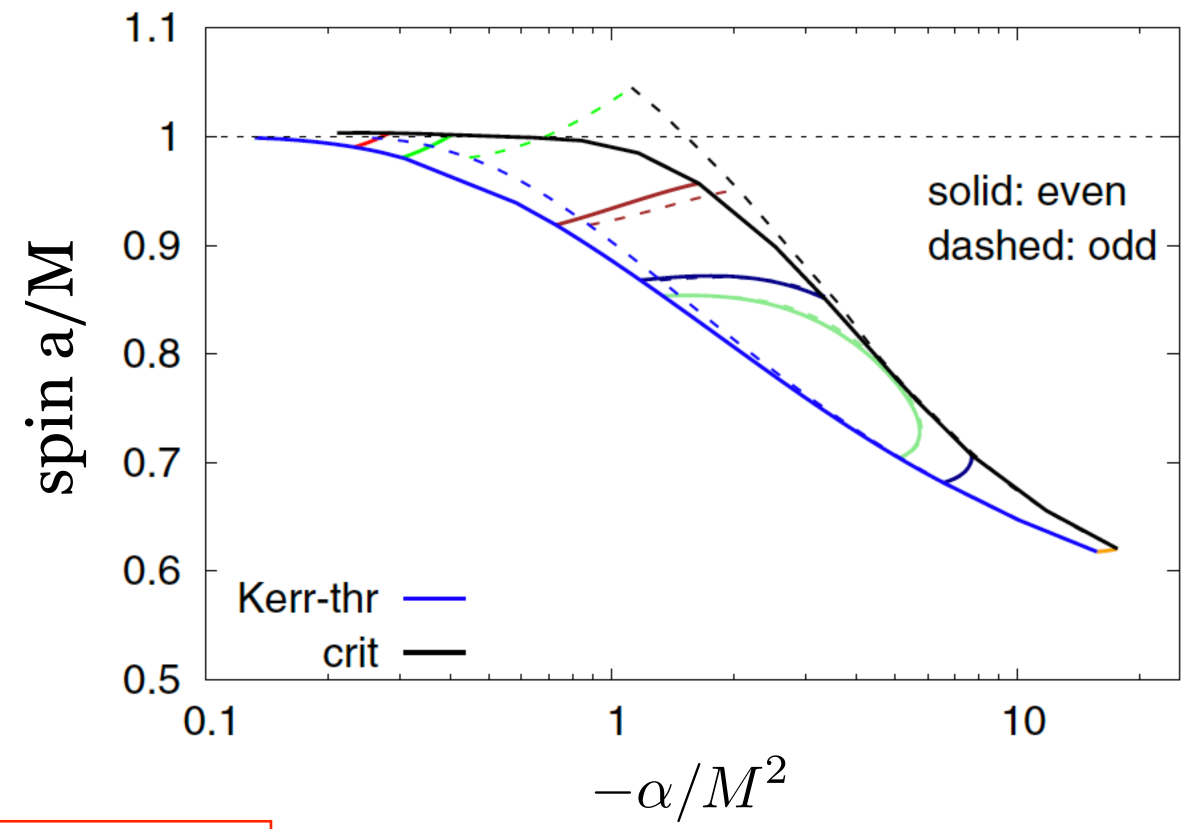
Evading the GW bounds (II)

- *Spin-induced scalarisation*: the non-GR BH is Kerr below some spin threshold. Above the threshold it undergoes a spontaneous "scalarisation", and becomes a non-Kerr BH with scalar "hair" (i.e. scalar charge).

- Scalarisation in EsGB gravity: it takes place for (see figure):

$$a \gtrsim 0.5M \quad \eta \sim -(0.1 - 10)$$

- Existing GW observations probe BHs with $a \lesssim 0.7M$, so they could miss higher spin scalarised systems.



Adapted from Berti et al. (2021). See also Herdeiro et al. (2021)

- M87* could be rapidly spinning, so scalarisation could be a viable way of evading the GW bounds. However, this possibility represents a small portion of the a/M-η parameter space.

Part I conclusions

- The combination of GW bounds and mass-suppression of dimensional coupling constants makes BH shadow-based tests of gravity blind to a large portion of non-GR theories.

- Shadows could still probe theories with dimensionless constants or special cases where screening and/or spin-scalarisation invalidate GW bounds.

- In all cases the uncertain accretion flow properties introduce a moderate error. The regime $b < b_{\mathrm{GR}}$ may not be affected.

- The results discussed here should have similar implications for astrometric tests of GR around our Galactic SgrA* supermassive BH (e.g. the GRAVITY experiment).

- The same can be said for electromagnetic observations of accretion flows around AGNs (since both the matter and radiation move along geodesics of the supermassive BH).

Part II
Quasi-normal modes of non-GR black holes

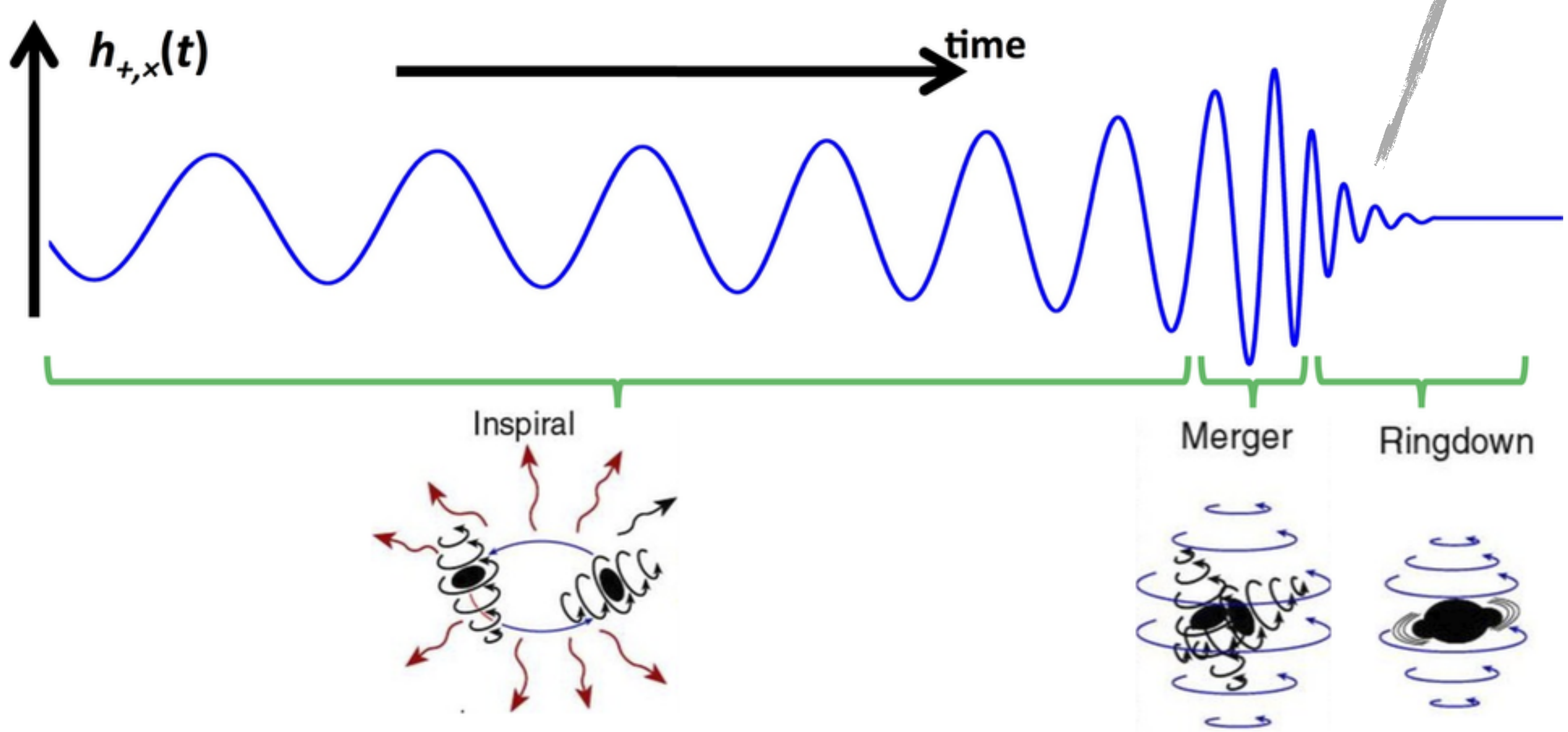
based on:

KG & Silva PRD (2019)
Silva & KG PRD (2020)
Bryant, Silva, Yagi & KG (in preparation)

BH ringdown and "spectroscopy"

$$\omega_n = \omega_R(M,a) + i\omega_I(M,a)$$

$$n = 0, 1, ..$$



BH spectroscopy beyond GR

- Testing deviations from GR: extraction of QNM frequencies from ringdown signal (“BH spectroscopy”).

- This approach requires modelling of QNMs of BHs beyond GR.

- The relevant QNM perturbation wave equations have been derived for a number of theories [or even classes of theories as in Tattersall et al. (2018)]. In their vast majority, these equations refer to spherically symmetric (i.e. non-rotating) BHs.

- Despite the relative wealth of perturbation equations, relatively few QNM solutions have been obtained so far.

- Our work on QNMs is based on the short-wavelength eikonal approximation as a means for solving the equations. The approach is largely “theory-agnostic”.

Geodesic analogy of QNMs

- In GR, the eikonal approximation is based on the geodesic analogy of the BH's fundamental QNM: its frequency $\omega = \omega_R + i\omega_I$ can be directly linked to the geodesic properties of the photon ring (i.e. the unstable circular orbit of photons).

- The key quantities are the orbital frequency and Lyapunov exponent (rate of diverge/convergence of light rays near the photon ring).

In a spherically symmetric spacetime:

Photon ring

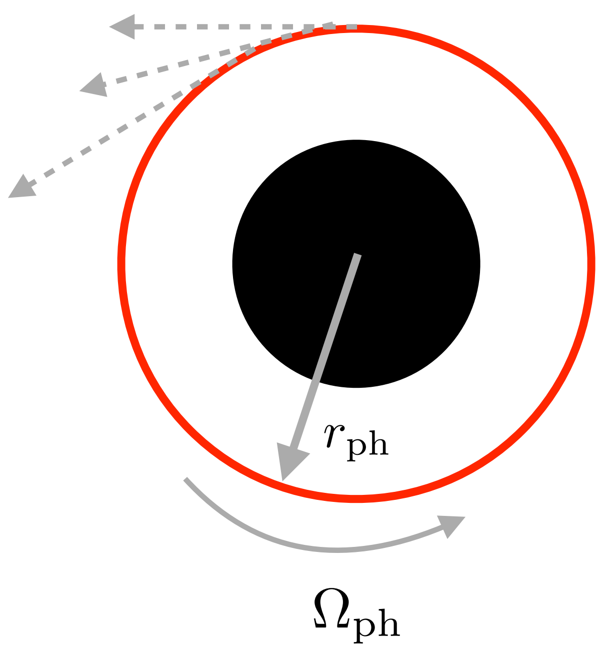
$$\Omega_{\rm ph} = \frac{u^\varphi}{u^t} = \frac{1}{b_{\rm ph}} = \frac{\sqrt{-g_{tt}(r_{\rm ph})}}{r_{\rm ph}}$$

$$\gamma_{\rm ph}^2 = -\frac{1}{2}\left[r^2 g_{tt}\frac{d^2}{dr^2}\left(\frac{g_{tt}}{r^2}\right)\right]_{\rm ph}$$

Eikonal QNM

$$\omega_R = \ell\Omega_{\rm ph}$$

$$\omega_I = -\frac{1}{2}|\gamma_{\rm ph}|$$



QNMs beyond GR

- In general, a non-GR theory will have the usual tensorial (metric) part coupled with a scalar field degree. This coupling is reflected in the perturbation equations describing QNMs. In spherical symmetry, and after separation of variables, these have the general form:

$$\frac{d^2\psi}{dx^2} + \left[\omega^2 - V_\psi(\ell, r)\right]\psi = a_0(\ell, r, \omega)\Theta + a_1(\ell, r, \omega)\frac{d\Theta}{dx}$$

$$\frac{d^2\Theta}{dx^2} + g(r)\frac{d\Theta}{dx} + \left[\omega^2 - V_\Theta(\ell, r)\right]\Theta = b_0(\ell, r, \omega)\psi + b_1(\ell, r, \omega)\frac{d\psi}{dx}$$

ψ= tensor perturbation

Θ= scalar perturbation

$x(r)$ = tortoise coordinate

- Eikonal approximation (ε=bookkeeping parameter):

$$\psi = A_\psi(x)e^{iS(x)/\epsilon} \qquad \Theta = A_\Theta(x)e^{iS(x)/\epsilon}$$

$$\omega = \mathcal{O}(\ell), \quad \ell = \mathcal{O}(\epsilon^{-1}), \quad \epsilon \ll 1$$

with the condition at the "peak"

$S_{,x} = 0$ at $r = r_{\rm m}$

BHs in Gauss-Bonnet gravity

- Einstein-scalar-Gauss-Bonnet gravity was discussed in Part I of this talk.

$$\mathcal{L} = \frac{1}{16\pi}\left[R - \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - \alpha f(\phi)\mathcal{R}^2_{\rm GB}\right] \qquad \mathcal{R}^2_{\rm GB} = R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} - 4R^{\alpha\beta}R_{\alpha\beta} + R^2$$

- We consider *non-rotating* BHs, assuming an expansion in $\dfrac{\alpha}{M^2} \ll 1$

$$ds^2 = -A(r)dt^2 + B^{-1}(r)dr^2 + r^2 d\Omega^2$$

$$A = 1 - \frac{2M}{r} - \frac{\alpha^2 f_0'^2}{M^4}\frac{M}{r}\left(\frac{49}{40} - \frac{M^2}{3r^2} - \frac{26M^3}{3r^3} - \frac{22M^4}{5r^4} - \frac{32M^5}{5r^5} + \frac{80M^6}{3r^6}\right) + \mathcal{O}(\alpha^4)$$

$$B = 1 - \frac{2M}{r} - \frac{\alpha^2 f_0'^2}{M^4}\frac{M}{r}\left(\frac{49}{40} - \frac{M}{r} - \frac{M^2}{r^2} - \frac{52M^3}{3r^3} - \frac{2M^4}{r^4} - \frac{16M^5}{5r^5} + \frac{368M^6}{3r^6}\right) + \mathcal{O}(\alpha^4)$$

$$\phi_0 = \frac{2\alpha f_0'}{M^2}\frac{M}{r}\left(1 + \frac{M}{r} + \frac{4M^2}{3r^2}\right) + \frac{\alpha^2 f_0' f_0''}{M^4}\frac{M}{r}\left(\frac{73}{30} + \frac{73M}{30r} + \frac{146M^2}{45r^2} + \frac{73M^3}{15r^3} + \frac{224M^4}{75r^4} + \frac{16M^5}{9r^5}\right) + \mathcal{O}(\alpha^3)$$

where $f_0' = f'(\phi = 0)$

Axial perturbations & QNMs

- The axial parity modes do *not* couple to the scalar field. Their radial eigenfunction Q is described by the equation:

$$\frac{d^2Q}{dx^2} + p_{\rm ax}(r)\frac{dQ}{dx} + \left[\omega^2 - V_{\rm ax}(r)\right] Q = 0$$

- Eikonal limit ansatz: $Q(x) = A_{\rm ax}(x) e^{iS(x)/\epsilon}$

- Solving the equation to $\mathcal{O}(\epsilon^{-1})$ and $\mathcal{O}(\epsilon^{0})$ order leads to ω_R and ω_I. As it turns out, the peak is located at $r_{\rm m} = r_{\rm ph} = 3M$.

- Leading-order eikonal results, accurate to $\mathcal{O}(\alpha^2)$

$$M\omega_R = \frac{\ell}{3\sqrt{3}}\left(1 - \frac{71987}{174960}\frac{\alpha^2 {f_0'}^2}{M^4}\right) \qquad M\omega_I = -\frac{1}{6\sqrt{3}}\left(1 - \frac{121907}{174960}\frac{\alpha^2 {f_0'}^2}{M^4}\right)$$

- The axial eikonal modes admit the same geodesic analogy as in GR.

Polar perturbations & QNMs

- In contrast to the previous case, the polar parity modes are described by coupled equations for the tensor and scalar perturbations.

$$\frac{d^2\psi}{dx^2} + p_{\rm pol}(r)\frac{d\psi}{dx} + \left[A_{\rm pol}(r)\omega^2 - V_\psi(\ell, r)\right]\psi = a_0(\ell, r, \omega)\Theta + a_1(\ell, r, \omega)\frac{d\Theta}{dx}$$

$$\frac{d^2\Theta}{dx^2} + \left[\omega^2 - V_\Theta(\ell, r)\right]\Theta = b_0(\ell, r, \omega)\psi + b_1(\ell, r, \omega)\frac{d\psi}{dx}$$

The potentials and coupling functions are derived to $\mathcal{O}(\alpha^2)$ precision

- An interesting technical detail: the eikonal limit ε << 1 and the expansion in the coupling α do not commute, i.e. it matters which expansion comes first and which parameter is formally the smallest. In order to ensure a smooth GR limit we take α << ε.

Polar QNMs

- We show leading-order eikonal results, accurate to $\mathcal{O}(\alpha^2)$

$$M\omega_{R\pm} = \frac{\ell}{3\sqrt{3}}\left(1 \pm \frac{4}{27}\frac{\alpha^2\ell^2 f_0'^{\,2}}{M^4}\right) \qquad M\omega_{I\pm} = -\frac{1}{6\sqrt{3}}\left(1 \mp \frac{44}{729}\frac{\alpha^2\ell^2 f_0'^{\,2}}{M^4}\right)$$

- As in the axial case, non-GR corrections first appear at $\mathcal{O}(\alpha^2)$. In addition, the polar modes displays a “Zeeman” splitting with two possible solutions.

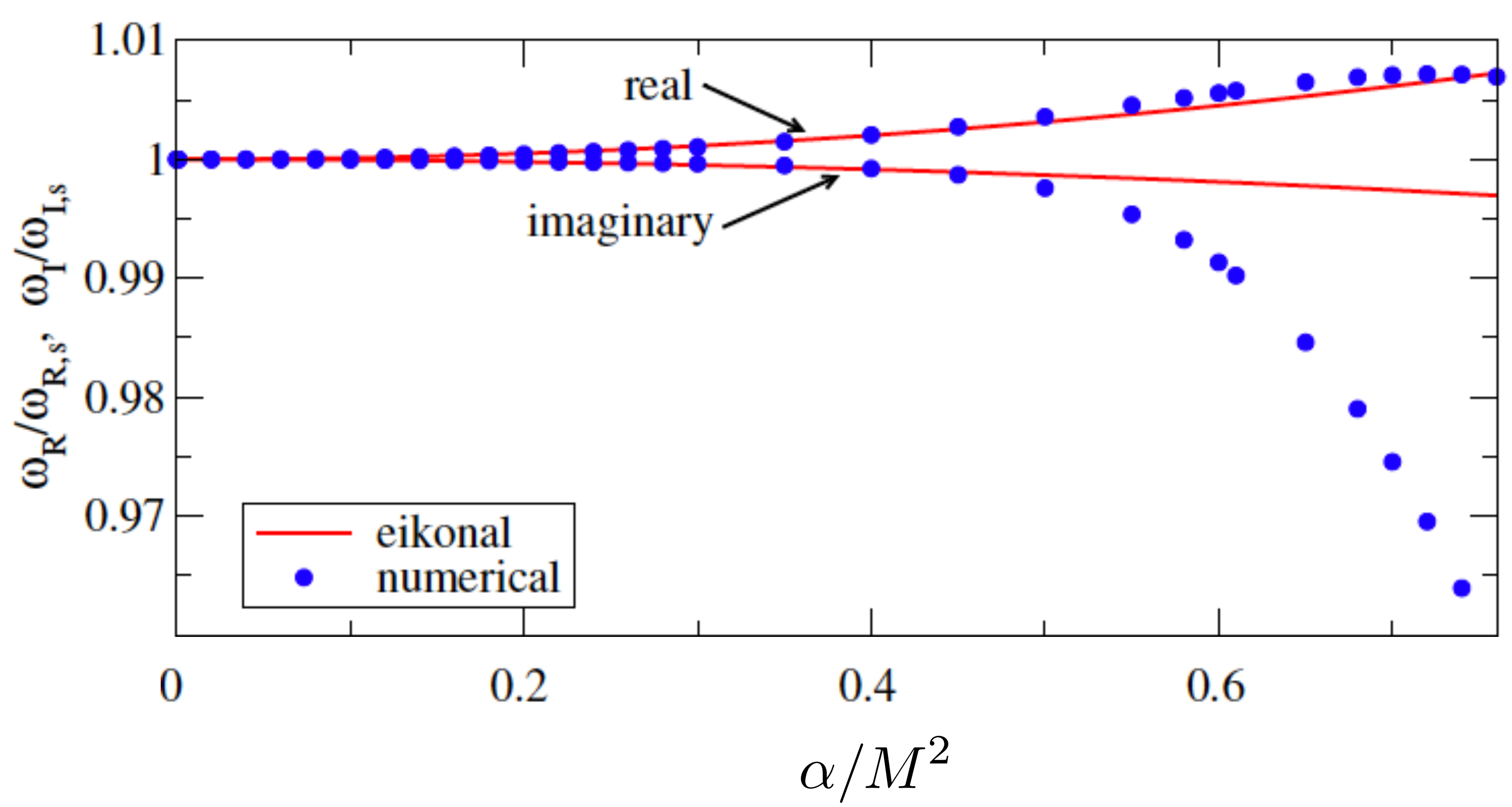
- It is unclear if the present case of coupled equations admits some sort of geodesic analogy.

Eikonal *vs* numerical QNMs (axial)

- The same QNMs have been calculated numerically (and to higher order in α) by Blázquez-Salcedo et al. (2016) for the special case of Einstein-dilaton-GB gravity, i.e.

$$f(\phi) = \frac{1}{4} e^{\phi}$$

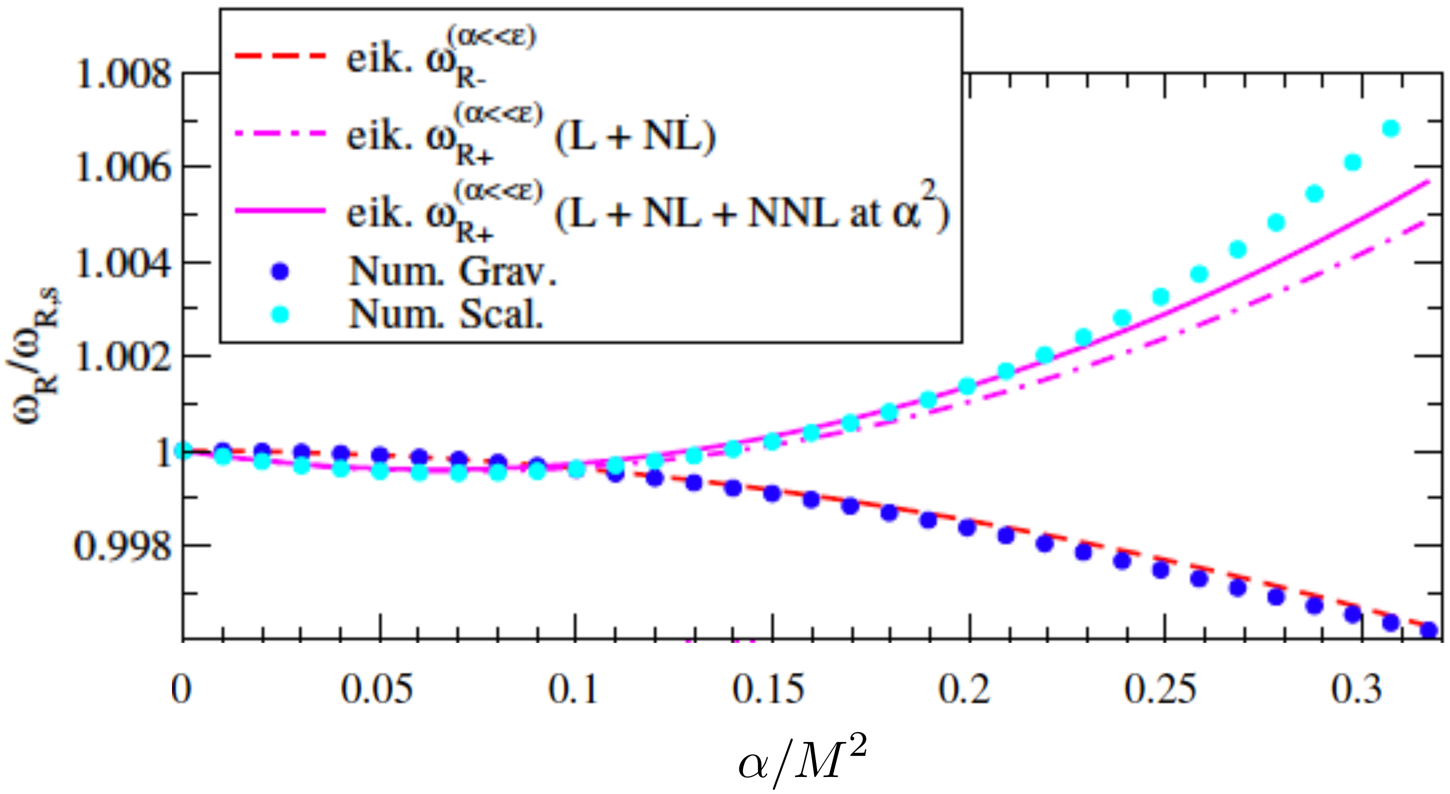
- The following figures show the *l*=2 QNM frequencies normalised to their corresponding GR values.

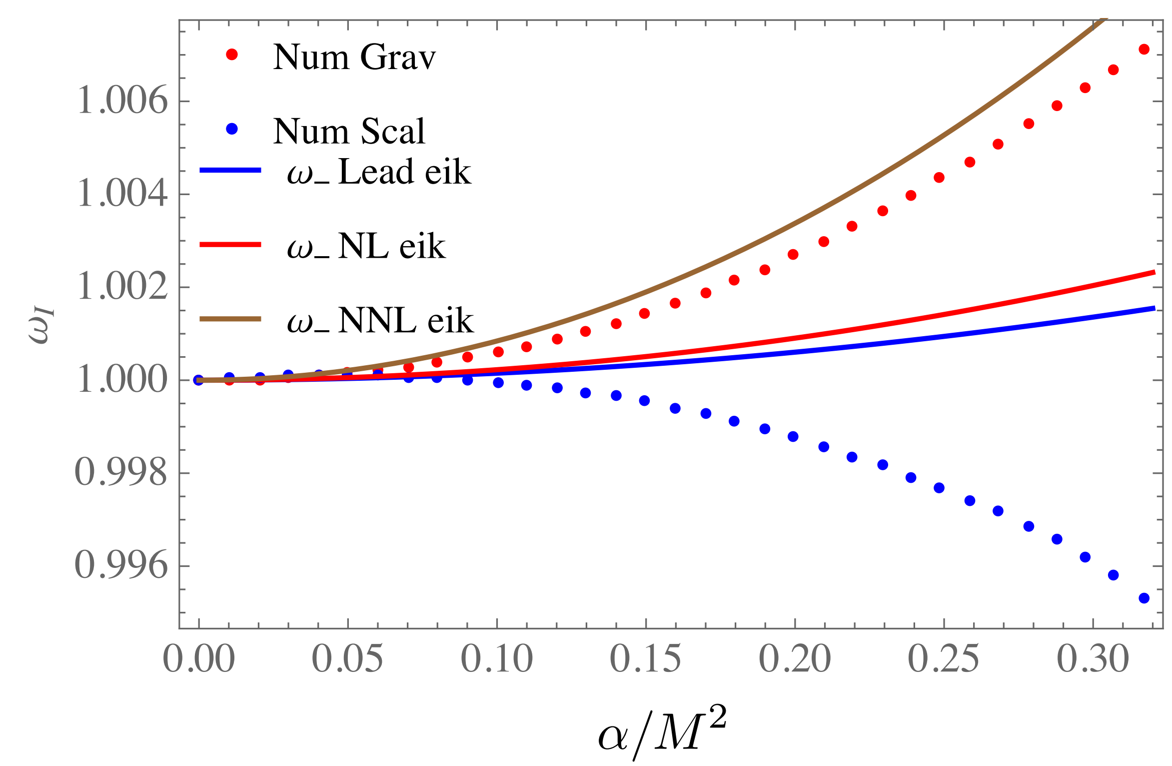


Eikonal *vs* numerical QNMs (polar)

- The polar QNMs have two branches, “gravitational-led” and “scalar-led”, named after their corresponding GR limit (α—>0). These two branches correspond to our +/- solutions.

- We plot leading order and higher order eikonal results.





Part II conclusions

- The eikonal approximation is a versatile tool for calculating QNMs of non-GR BHs. The example of EdGB (as well as previous work on Chern-Simons gravity) suggests a few % precision with respect to numerical data. This can be improved by adding higher-order pieces.

- The eikonal scheme should be equally well applicable to non-GR BHs with spin (in which case a numerical computation might not be that easy).

- No simple geodesic analogy was found for the coupled tensor-scalar QNMs (but this does not mean there isn't one!).

- The eikonal QNM formulae (and the wave equations themselves) are subject to the same mass-suppression effect of the coupling constant as the geodesic equations. LISA's ability to probe other theories may be compromised (this was first suggested in Maselli et al. (2020)).

Thank you for your attention

Σας ευχαριστώ για την προσοχή σας